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Methodology for Probabilistic Life Prediction of Multiple-Anomaly Materials

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Titanium gas-turbine engine components may contain anomalies that are not representative of nominal conditions. If undetected, they can lead to uncontained failure of the engine and associated loss of life. A probabilistic framework has been developed to predict the risk of fracture associated with titanium rotors and disks containing rare material anomalies. A recent Federal Aviation Administration Advisory Circular also provides guidance for the design of these components. However, some materials may exhibit relatively higher anomaly occurrence rates compared to those found in titanium alloys. In addition, the crack formation life for these materials may be nonnegligible and must be considered in the risk computation. When these materials are used, a single disk could contain a number of anomalies with unequal crack formation periods, and so the existing probabilistic framework is no longer valid. A methodology is presented for probabilistic life prediction of components with relatively large numbers of material anomalies. It is an extension of the probabilistic framework originally developed for titanium materials with hard alpha anomalies. The methodology is presented and illustrated for an aircraft gas-turbine engine disk. The results can be applied to fracture-mechanics-based probabilistic life prediction of alloys with large numbers of material anomalies.

Nomenclature

A	=	occurrence of anomaly in zone i
A_j	=	occurrence of j anomalies in zone
$\mathrm{d}a/\mathrm{d}N$	=	crack growth rate
$F_{i\|A}$	=	fracture failure event in zone i given anomaly in zone i
$F_{i\|A_j}$	=	fracture failure event associated with zone i given that j anomalies are present
$F_{i\|A,L}$	=	fracture event of component given that anomaly is placed at life limiting location
K_C	=	fracture toughness
$K_{i\|A,L}$	=	stress intensity factor associated with anomaly at life limiting location in zone i
N	=	crack growth life
p_A	=	probability associated with event A
p_i	=	probability of fracture of zone i
$p_{i\|A}$	=	probability associated with event $F_{i\|A}$
$p_{i\|A,L}$	=	probability associated with event $F_{i\|A,L}$
V	=	component, volume
V_i	=	volume of zone i
λ_i	=	mean anomaly occurrence rate associated with zone i
λ_V	=	mean anomaly occurrence rate associated component volume

Introduction

THE materials used in aircraft gas-turbine engine rotating components may occasionally contain anomalies introduced during the manufacturing process that are not representative of nominal conditions. For example, the premium-grade titanium materials used for fan and compressor disks may contain brittle (hard alpha) anomalies that form during the triple vacuum arc melting process. These anomalies may occur anywhere within a billet and may change shape during the forging process. If undetected during manufacturing or subsequent field inspection, they can ultimately lead to uncontained failure of the engine. The consequences of this event can be catastrophic, including loss of life and loss of the aircraft.[1]

A recent Federal Aviation Administration Advisory Circular[2] provides guidance on the assessment of the risk of fracture associated with inherent anomalies in high-energy rotating components. Reference 2 describes a probabilistic damage tolerance process that can be used to predict the probability of fracture associated with titanium rotors, and establishes a design target risk for this event. This document was developed for titanium materials with hard alpha anomalies. The occurrence of anomalies in this material is considered to be a relatively rare event.

A probabilistic framework was previously developed to predict the risk of fracture associated with rotors and disks containing rare material anomalies.[3,4] It addresses the influences of the primary random variables such as initial anomaly size, applied stress, and fracture-mechanics-related material variables. The framework was originally developed for titanium materials, where anomalies are assumed to form growing cracks during the first cycle of applied load. Because the anomaly occurrence rate associated with titanium is extremely small, component failure probability can be approximated as the sum of the failure probabilities of subregions (zones) of approximately equal risk.[5]

However, some materials may exhibit relatively higher anomaly occurrence rates compared to those found in premium-grade titanium alloys. For these materials, estimating the component failure probability as the sum of the failure probabilities of individual subregions may result in overconservative risk predictions. In addition, the crack formation life (i.e., the number of cycles required for an anomaly to form a growing crack) associated with these materials may be nonnegligible. The crack formation life is a nondeterministic variable that must also be considered in the risk computations. When multiple anomaly materials are used, a single component could contain a number of anomalies with random crack formation periods, and so the existing probabilistic framework is no longer valid.

Several engineering models have been proposed over the past decade to address materials with high anomaly occurrence rates. Some are focused primarily on characterization of the anomaly occurrence rate,[6] whereas others consider variables such as the size

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and location of the anomalies with respect to the free surfaces of a component.[7,8] Comprehensive design systems have been developed that address these variables and additional factors such as the consequences of component failure.[9] In addition, some researchers have considered multiple failure modes associated with these materials.[10]

In this paper, a more general methodology is presented for probabilistic life prediction of components with relatively large numbers of material anomalies. The methodology, based on the probabilistic framework originally developed for titanium materials with hard alpha anomalies,[11] is applicable to other alloys. The methodology is presented and illustrated for an aircraft gas-turbine engine disk. The results can be applied to fracture-mechanics-based probabilistic life prediction of alloys with large numbers of material anomalies.

Risk Assessment of Rare Anomaly Materials

In Ref. 2 it is recommended that a number of variables be considered for a probabilistic risk assessment of disk fracture, including initial anomaly size, component stress and volume, material properties, crack propagation life, inspection probability of detection (POD), and shop visit time. Many of these variables can be modeled parametrically using data from established sources. For example, Ref. 2 provides a number of anomaly distributions for titanium materials that are based on industry melting, forging, and inspection processes. It also provides industry-developed data for modeling the inspection POD random variable. The main descriptors of the shop visit variable can be estimated from maintenance records associated with a fleet of aircraft.

A probabilistic framework[3,4] was developed previously to quantify the risk of fracture associated with rare anomaly materials. It provides probabilistic treatment of the variables specified in Ref. 2 and introduces two additional random variables to address the variabilities associated with applied stress and crack propagation life values. The random variables considered in the framework are shown in Fig. 1.

The computation time associated with a probabilistic fracture-mechanics-based risk assessment may be nontrivial. To improve the efficiency of risk computations, an approximate solution is used to address the uncertainty associated with the location of the anomaly. As shown in Fig. 2, the disk is subdivided into regions of approximately equal risk, called zones. The volume of material contained in a single zone will experience similar stresses, material properties, inspection schedules, and POD, and, therefore, an initial anomaly located anywhere in the zone will exhibit a similar crack growth life. The crack propagation life is estimated using stress intensity factor solutions for cracks in rectangular plates[12] that approximate the actual component geometry and stress distributions (Fig. 3). For rare anomaly materials it is assumed that a zone has no more than one significant anomaly. To ensure a conservative risk estimate, the anomaly is placed in a location within the zone that minimizes the crack growth life values, also known as the life limiting location of the zone (Fig. 2).

For rare anomaly materials, the probability of fracture is dependent on the size and the location of the anomaly within a component. Within a zone, the probability of fracture p_i is given by

$$p_i = P\left(F_{i|A} \bigcap A\right) \tag{1}$$

Events $F_{i|A}$ and A are statistically independent, and so Eq. (1) can be expressed as

$$p_i = p_{i|A} \cdot p_A \tag{2}$$

Within a zone, the conditional probability of fracture is bounded by the probability of fracture at the life limiting location:

$$P(F_{i|A}) \leq P(F_{i|A,L}) \tag{3}$$

or

$$p_{i|A} \leq p_{i|A,L} \tag{4}$$

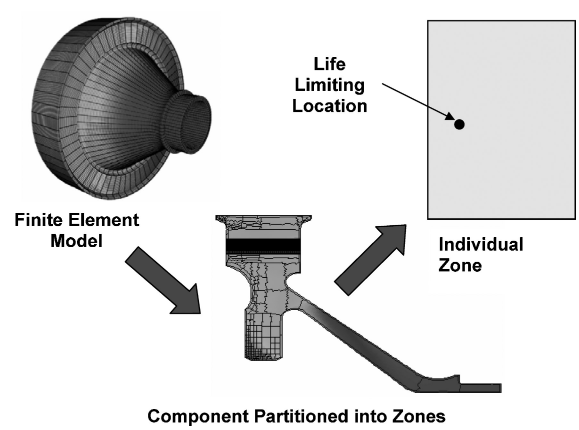

Fig. 2 Zone-based approach used for risk assessment of rare anomaly materials in which a component is partitioned into subregions of approximately equal risk.

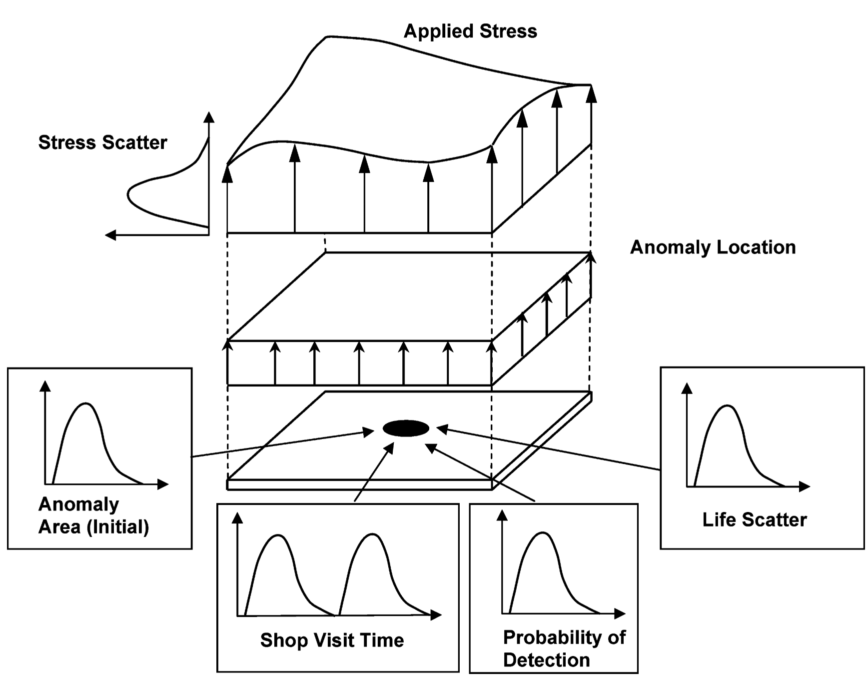

Fig. 1 Random variables associated with risk assessment of rare anomaly materials.

Equation (4) provides an upper-bound estimate for the conditional probability of fracture associated with a zone. As the number of zones is increased, the value of $p_{i|A}$ approaches $p_{i|A,L}$. Note that p_A can be modeled as a Poisson process (see Ref. 13) with a mean anomaly occurrence rate λ_i that is proportional to the volume of material in the zone:

$$\lambda_i = (V_i/V)\lambda_V \tag{5}$$

The probability of fracture for a component is modeled as a series system of the m zones[5,11]:

$$p_F = P\left[F_1 \cup F_2 \cup \cdots \cup F_m\right] = 1 - P\left[\bigcap_{i=1}^{m} \bar{F}_i\right] \tag{6}$$

If the mean anomaly occurrence rate is relatively small, the probability of more than one significant anomaly in the component is negligible, and Eq. (6) reduces to[14]

$$p_F \simeq \sum_{i=1}^{m} p_i \tag{7}$$

Risk Assessment of Multiple Anomaly Materials

The anomaly occurrence rate is highly dependent on the condition of the raw material and the manufacturing process used to shape the material into the final component shape.[15] Some materials may exhibit anomaly rates that are significantly greater than those associated with titanium. Therefore, some zones may contain multiple anomalies (Fig. 4).

If it is assumed that there are no physical interactions among anomalies in a zone, then the crack propagation life associated with each anomaly could be estimated using the rectangular plate stress intensity factor solutions (Fig. 3). However, the computation time associated with this approach for multiple anomalies could become unmanageable for the reasons cited in the preceding section. Zone risk can instead be estimated by placing the multiple anomalies at the life limiting location (Fig. 4). This provides a conservative (upper bound) risk estimate that converges to the true solution as the number of zones becomes large.[16]

Some anomaly types, for example, titanium hard alpha, have negligible formation lives and form growing cracks almost immediately.[11] However, in other materials, the crack formation life may be nonnegligible and must be considered in the risk computation.[7,17] A crack formation random variable can, therefore, be introduced to address the variability associated with the number of cycles required for an anomaly to form a growing crack.

Consider a zone that has multiple anomalies. The probability of fracture within the zone can be expressed as

$$p_i = P\left[\begin{array}{c}\left(F_{i|A_1} \bigcap A_1\right) \bigcup \left(F_{i|A_2} \bigcap A_2\right) \cdots \\ \bigcup \left(F_{i|A_{n-1}} \bigcap A_{n-1}\right) \bigcup \left(F_{i|A_n} \bigcap A_n\right)\end{array}\right] \tag{8}$$

or

$$p_i = P\left[1 - \bigcap_{j=1}^{n} \left(\bar{F}_{i|A_j} \bigcap A_j\right)\right] \tag{9}$$

It follows that

$$\bar{p}_i = P(\bar{F}_i) = P\left[\bigcap_{j=1}^{n} \left(\bar{F}_{i|A_j} \bigcap A_j\right)\right] \tag{10}$$

Events A_j are mutually exclusive and collectively exhaustive. If events $F_{i|A_j}$ and A_j are independent, then Eq. (9) can also be expressed as

$$p_i = \sum_{j=1}^{n} P\left(F_{i|A_j}\right) \cdot P(A_j) \tag{11}$$

When it is noted that the probability of fracture of a component is equal to the probability union of the zones, and Eq. (9) is substituted

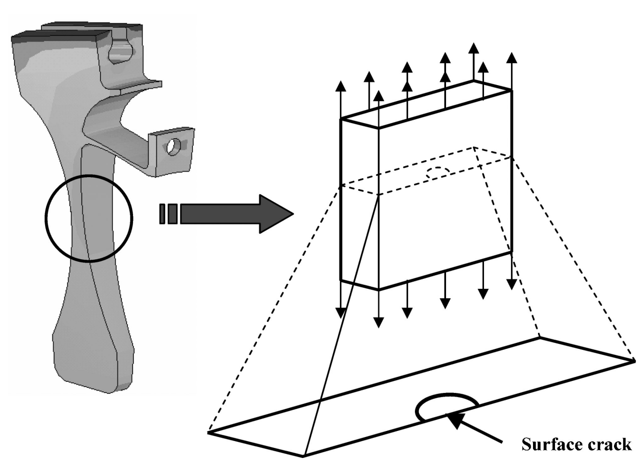

Fig. 3 Crack propagation life estimated using stress intensity factor solutions for cracks in rectangular plates.

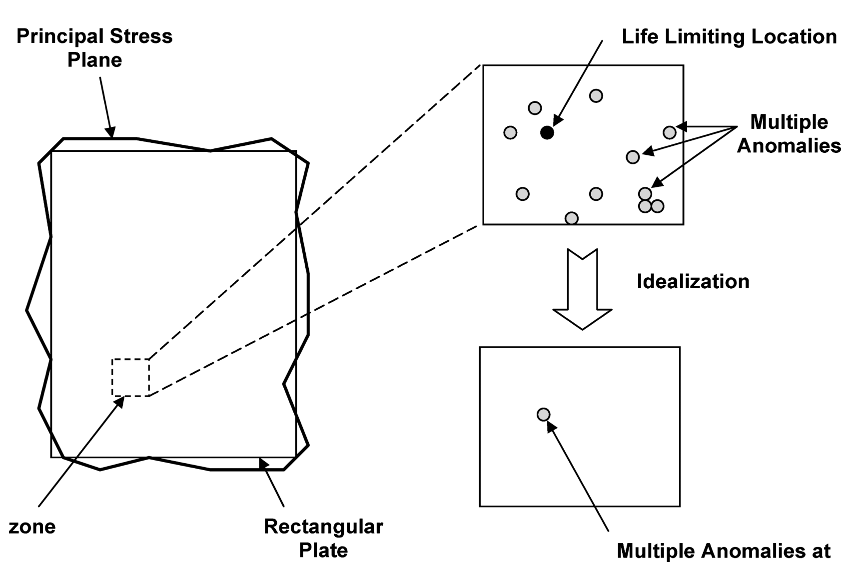

Fig. 4 Multiple anomalies placed at life limiting location of zone for probabilistic fracture mechanics computations.

into Eq. (6), the following expression is obtained:

$$p_F = 1 - P\left[\bigcap_{i=1}^{m}\left(\bigcap_{j=1}^{n}\bar{F}_{i|A_j}\bigcap A_j\right)\right] \tag{12}$$

If zone failures are treated as independent events, Eq. (12) becomes

$$p_F = 1 - \prod_{i=1}^{m}(1 - p_i) \tag{13}$$

Zone failures are generally partially correlated due to the inertia loading-based stress values that are simultaneously applied to all zones. Equation (13) provides a conservative estimate of the component failure probability (provided that correlation among zone failures is nonnegative). Note that if the number of anomalies is relatively small, Eq. (13) can be approximated using Eq. (7).

Multiple Anomalies at Life Limiting Location

When multiple anomalies are present, the conditional probability of fracture of a zone can be modeled as a series system consisting of the failure associated with each anomaly:

$$P\left(F_{i|A_j}\right) = 1 - P\left(\bigcap_{j=1}^{n}\bar{F}_{i|A_j}\right) \tag{14}$$

As noted earlier, $F_{i|A_j}$ has an upper bound value associated with the life limiting location as specified in Eq. (4). If it is assumed that all anomalies are at the life limiting location, Eq. (14) becomes

$$P\left(F_{i|A_j}\right) \le 1 - \left[P\left(\bar{F}_{i|A_j,L}\right)\right]^j \tag{15}$$

Poisson Distributed Anomalies

As noted earlier, an anomaly occurrence can be modeled as a Poisson distribution:

$$P(A_j) = \left[(\lambda_i)^j / j!\right]\exp(-\lambda_i) \tag{16}$$

When the expressions in Eqs. (15) and (16) are substituted into Eq. (11), the probability of fracture for a zone becomes

$$p_i = \sum_{j=1}^{n}\left(\left\{1 - \left[P\left(\bar{F}_{i|A_j,L}\right)\right]^j\right\}\cdot\frac{(\lambda_i)^j}{j!}\exp(-\lambda_i)\right) \tag{17}$$

which can also be expressed as[9]

$$p_i = 1 - \exp[-\lambda_i \cdot p_{i|A,L}] \tag{18}$$

When this expression for p_i is substituted in Eq. (13), the component probability of fracture becomes

$$p_F = 1 - \prod_{i=1}^{m}\exp[-\lambda_i \cdot p_{i|A,L}] \tag{19}$$

where zone failure occurs when the stress intensity factor at the life limiting location $K_{i|A,L}$ exceeds the fracture toughness K_C:

$$p_{i|A,L} = P(K_{i|A,L} > K_C) \tag{20}$$

General Probabilistic Framework

In addition to inherent anomalies, engine disks may also be subjected to induced anomalies. These anomalies are introduced during manufacturing and handling operations and are typically found on machined surfaces. Several researchers have reported disk failures that initiated at the interior surfaces of bolt holes,[18,19] including some that have led to uncontained engine failures.[20] Inherent and induced anomalies are associated with different manufacturing processes, and so risk assessment is often treated separately for these two anomaly types.

However, there are also many similarities in the risk assessment of components with inherent and induced anomalies. Most of the random variables are identical for these anomaly types, for example, applied stress, crack growth life variability, inspection time, and POD. Life prediction for both anomaly types requires descriptions of crack geometry and associated boundary conditions, applied stresses on the crack plane, and fatigue crack growth properties. Although the specific stress intensity factor solutions may differ, for example, induced anomalies located at bolt holes requiring special treatment, the growth process is often very similar for these two anomaly types.

For component risk computations, Eq. (19) is valid for both inherent and induced anomalies. For inherent anomalies, risk is computed for zones using a volume-based anomaly occurrence rate. For induced anomalies, risk is computed for features using an area-based anomaly occurrence rate. When appropriate, the random crack formation life (and associated variables) is included in the probabilistic assessment.

This approach is shown in Fig. 5 (Ref. 21). It incorporates features that are common to these anomaly types into a general probabilistic

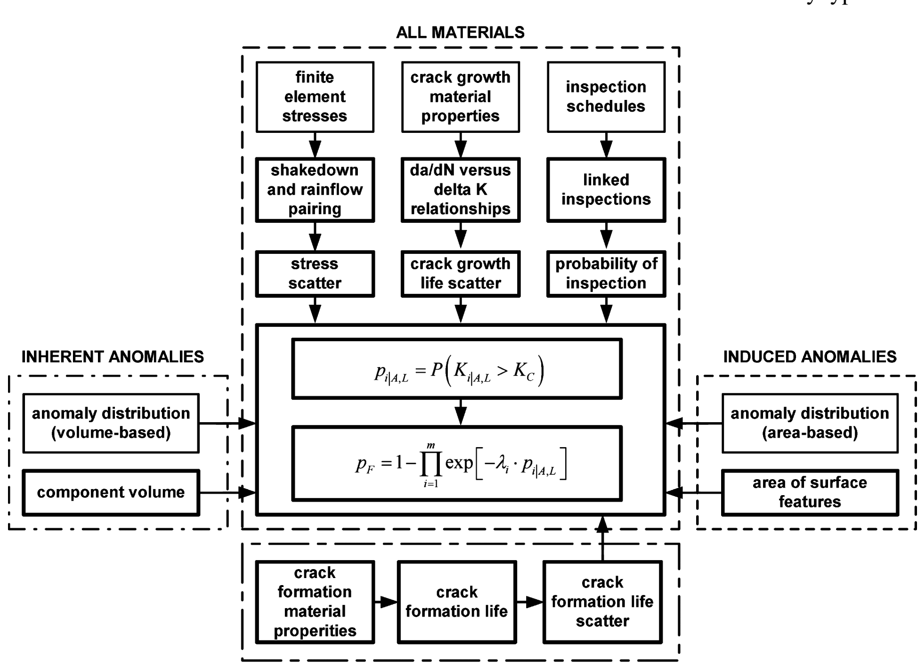

Fig. 5 General probabilistic framework for risk prediction of components with inherent and induced material anomalies.

framework and addresses the additional variables associated with nonzero crack formation times. The major advantage to this approach is that it reduces the number of redundant computational algorithms that must be developed and maintained to support risk computations.

Application to Gas-Turbine Engine Components

The probabilistic methodology for multiple anomaly materials is shown for the aircraft gas-turbine engine compressor disk shown in Fig. 6. Internal stresses and temperatures are based on finite element analysis results associated with four critical load steps in the flight cycle. The disk has a design life of 20,000 flight cycles. The disk is subdivided into 31 zones to account for the uncertainty associated with anomaly location.

Deterministic crack growth life is computed using stress intensity factor solutions for cracks in rectangular plates. The anomalies are placed at the life limiting location of the zone. The coordinates of the life limiting location can be identified by reviewing the life values associated with various placements of the anomaly within the zone. For example, the normalized life contours associated with zone 23 shown in Fig. 7 reveal the life limiting location that can be used for the probabilistic computations.

Zone probability of fracture values were computed using a combined technique of numerical integration and importance sampling.[22] Numerical results for the 10 most risk critical zones are indicated in Table 1. The zone conditional probability of fracture $p_{i|A}$ values quantify the risk associated with an anomaly placed at the life limiting location and are identical for both rare and multiple anomaly materials. However, the anomaly occurrence rates are significantly different for these two anomaly types, as are the associated zone unconditional probability of fracture p_i values.

For rare anomaly materials, it is assumed that there is no more than one significant anomaly in the component, and so component risk can be computed using Eq. (7). On the other hand, anomalies are more plentiful for multiple-anomaly materials. It becomes increasingly likely that more than one significant anomaly will be present in the disk, and so Eq. (7) is no longer valid. Therefore, component risk is computed using Eqs. (13) and (17) for a finite number of anomalies or Eq. (19) for an infinite number of anomalies.

Normalized risk values for multiple-anomaly materials based on the use of Eqs. (13) and (17) are shown in Fig. 8 for a number of anomaly occurrence rate values. For this example, it can be observed that the number of anomalies considered in Eq. (17) has a negligible influence on component risk results for zone anomaly occurrence rates less than about 10^{-3}. In fact, if the anomaly occurrence rate is small enough, the approximate model presented in Eq. (7) can be used for risk predictions. In Fig. 9, normalized risk results[23] are

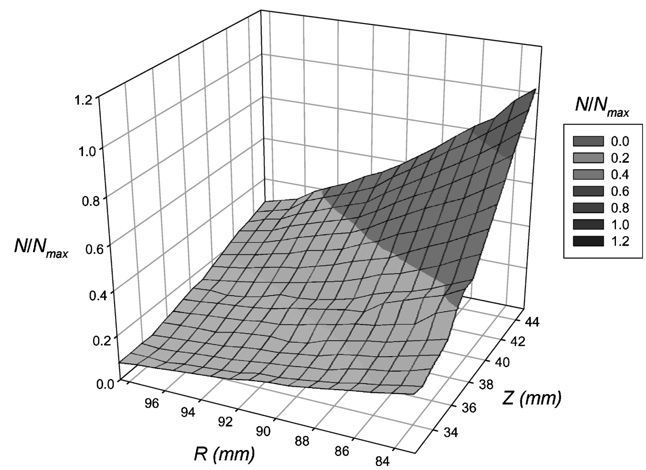

Fig. 7 Normalized life contour plot revealing coordinates of life limiting location associated with zone 23 of engine disk.

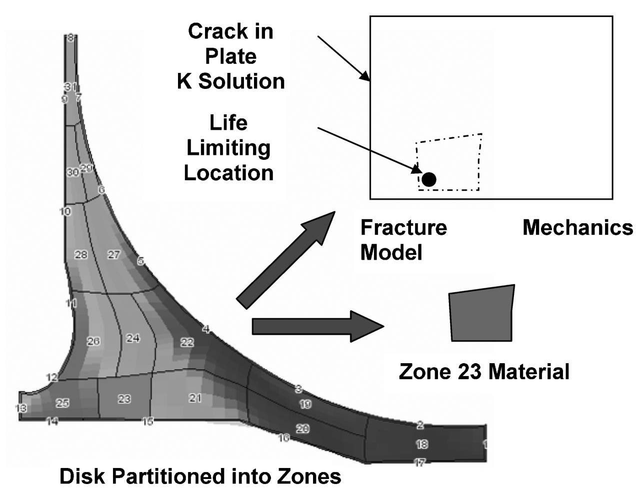

Fig. 6 Probabilistic methodology for multiple anomalies for gas-turbine engine compressor disk.

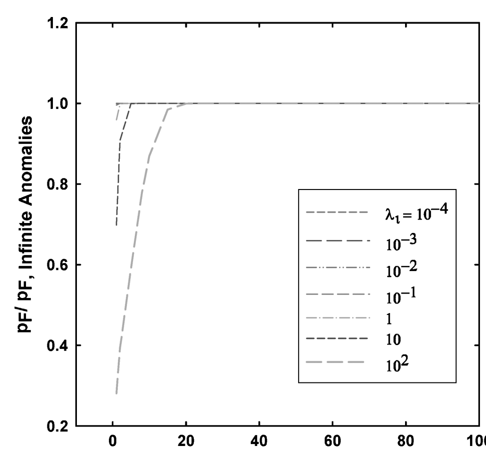

Fig. 8 Influence of anomaly occurrence rate λ_i and number of anomalies on normalized zone risk.

Table 1 Zone risk values associated with risk critical zones for gas-turbine engine component

		Rare anomalies		Multiple anomalies		p_i/p_F,
Zone	$p_{i\|A}$	λ_i	p_i	λ_i	p_i	%
10	$3.29E{-03}$	$5.31E{-06}$	$1.75E{-08}$	2.66	$8.75E{-03}$	4.3
11	$2.19E{-02}$	$3.05E{-06}$	$6.67E{-08}$	1.52	$3.34E{-02}$	16.3
12	$1.95E{-02}$	$1.94E{-06}$	$3.79E{-08}$	$9.72E{-01}$	$1.90E{-02}$	9.3
14	$2.37E{-02}$	$1.17E{-06}$	$2.77E{-08}$	$5.84E{-01}$	$1.39E{-02}$	6.8
21	$1.00E{-04}$	$5.32E{-05}$	$5.32E{-09}$	$2.66E{+01}$	$2.66E{-03}$	1.3
23	$4.02E{-04}$	$2.76E{-05}$	$1.11E{-08}$	$1.38E{+01}$	$5.55E{-03}$	2.7
25	$3.51E{-03}$	$3.16E{-05}$	$1.11E{-07}$	$1.58E{+01}$	$5.55E{-02}$	27.2
26	$1.40E{-03}$	$6.92E{-05}$	$9.69E{-08}$	$3.46E{+01}$	$4.85E{-02}$	23.8
28	$2.00E{-04}$	$5.80E{-05}$	$1.16E{-08}$	$2.90E{+01}$	$5.80E{-03}$	2.8
30	$2.99E{-04}$	$3.58E{-05}$	$1.07E{-08}$	$1.79E{+01}$	$5.35E{-03}$	2.6

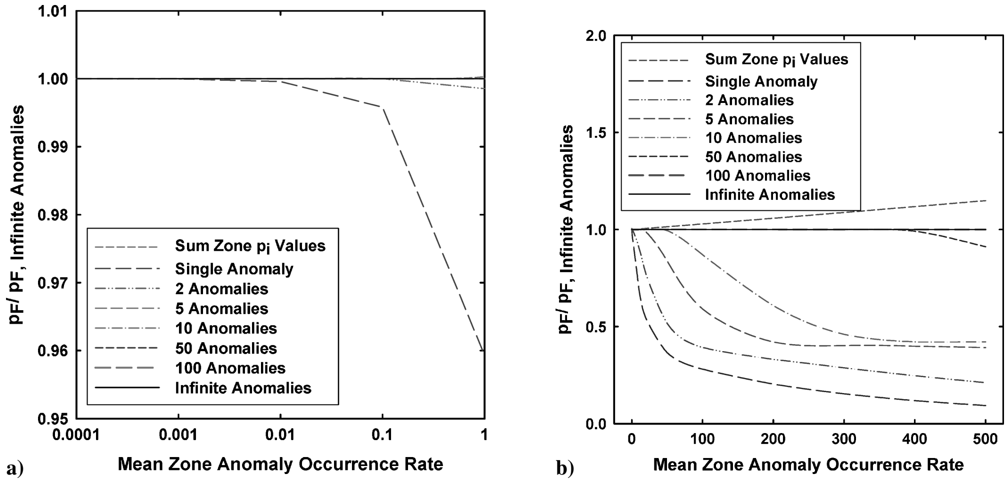

Fig. 9 Influence of number of anomalies on normalized component risk values: a) relatively rare anomalies and b) relatively frequent anomalies.

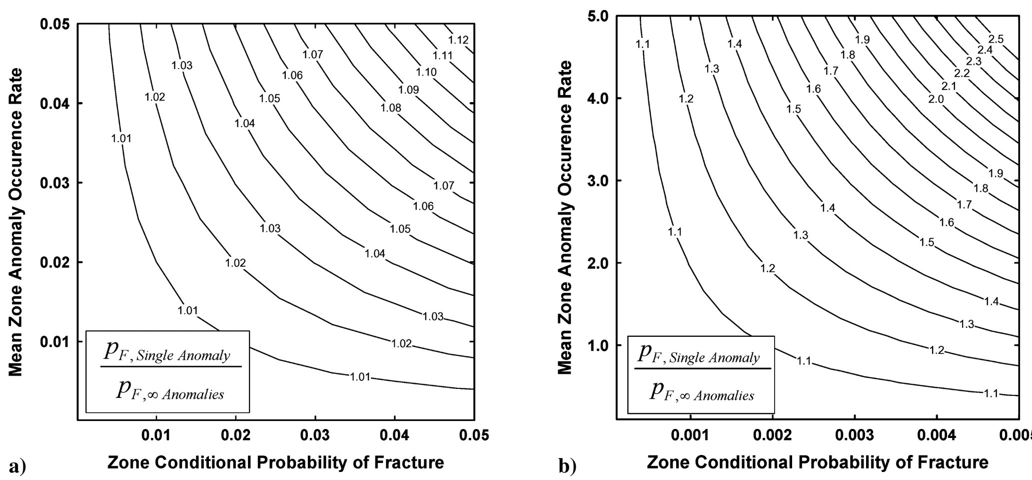

Fig. 10 Influence of zone anomaly occurrence rate λ_i and conditional probability of fracture $p_{i|A}$ on normalized component risk for $p_{i|A}$ values with relative magnitudes a) 10^{-2} and b) 10^{-3}.

shown for Eq. (7) and Eqs. (13) and (17), where it can be observed that the component risk results are nearly identical when the mean zone anomaly occurrence rate is less than 10^{-3}. Because the zone anomaly occurrence rate for titanium ranges from 10^{-7} to 10^{-5}, both of these approaches are valid for this anomaly type. Even for relatively high occurrence rates, the results converge to the infinite anomaly solution [Eq. (19)], provided that enough anomalies are considered in Eq. (17).

The results shown in Figs. 8 and 9 are limited to the range of zone conditional probability of fracture values associated with the compressor disk example. To generalize the results, consider a fictitious component consisting of 100 zones that have identical λ_i and $p_{i|A}$ values. For this component, the ratio of the component risk computed using Eqs. (7) and (19) can be expressed as[23]

$$\frac{\sum_{i=1}^{m} p_{i|A,L} \cdot p_A}{1 - \prod_{i=1}^{m} \exp(-\lambda_i \cdot p_{i|A,L})} \simeq \frac{m\lambda_i \, p_{i|A,L}}{1 - \exp(-\lambda_i \cdot p_{i|A,L})^m} \tag{21}$$

The influences of anomaly occurrence rate and conditional probability of fracture on component risk evaluated using Eq. (21) is shown in Fig. 10. If $p_{i|A}$ is relatively large (Fig. 10a), the results of Eqs. (7) and (19) are similar, except when λ_i is on the order of 10^{-2} or greater. Similarly, if $p_{i|A}$ is reduced somewhat (Fig. 10b), λ_i must have a value of about one or greater to influence the results.

Summary

A methodology was presented for probabilistic life prediction of multiple-anomaly materials. It is valid for both inherent and induced anomalies placed at the life limiting location of zones. The methodology was illustrated for an aircraft gas-turbine engine component, including the relative influences of the zone anomaly occurrence rate and conditional probability of fracture on overall component risk. The results can be used to extend the existing zone-based probabilistic framework for titanium materials (with hard alpha anomalies) for application to other alloys with a potentially large number of material anomalies.

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